Maximum/Minimum Word Problems: Vertex Form

There are two types of max/min word problems that we cover in MCF3M:

- Revenue Problems
- Geometry (Max Area) Problems

Revenue Problems

Remember, the y-value of the vertex is either the maximum or the minimum value the function can have.

Example 1

A small business’ profits over the last year have been related to the price of the only product. The relationship is

\[ R(x) = -0.4x^2 + 64x - 2400 \]

where \( R \) is the revenue measured in dollars and \( x \) is the price of the product measured in dollars.

What price would maximize the revenue?

To solve this we need to find the vertex! To find the maximum, (at the vertex) we need to complete the square.

Solution:

\[
R(x) = -0.4x^2 + 64x - 2400 \\
R(x) = -0.4(x^2 - 160x) - 2400
\]
Example 2

Amir sells his extreme cheesesteak sandwiches for $8 each. At this price he usually sells 100 cheesesteak sandwiches a day. After an informal survey, he concludes that for every $0.50 increase, he will sell five fewer sandwiches.

a) What price should Amir charge to maximize his revenue?

b) What is his maximum revenue?

Solution

a)

Let \( x \) = the number of $0.50 price increases

Let \( R(x) \) = the total revenue

Selling Price = \( 8 + 0.50x \)

Number of sandwiches sold = 100-5x

Revenue, \( R(x) = \text{Price} \times \text{Number sold} \)

\[ R(x) = (8 + 0.50x)(100-5x) \]

\[ R(x) = 800 - 40x + 50x - 2.5x^2 \]

\[ R(x) = -2.5x^2 + 10x + 800 \]

\[ R(x) = -2.5(x^2 - 4x) + 800 \]

\[ R(x) = -2.5(x^2 - 4x + (4)^2 - (4)^2) + 800 \]

\[ R(x) = -2.5(x^2 - 4x + 4 - 4) + 800 \]

\[ R(x) = -2.5(x^2 - 4x + 4) + 800 \]

\[ R(x) = -2.5(x - 2)^2 + 800 \]

The maximum revenue is $13,600
R(x) = -2.5(x - 2)^2 + 10 + 800
R(x) = -2.5(x - 2)^2 + 810

The function reaches a maximum when x = 2. So there should be two price increases to maximize the revenue.

Selling Price = 8 + 0.5x

= 8 + 0.5(2)

= $9

b) What is his maximum revenue?

We completed the square to put the function in vertex form.

R(x) = -2.5(x - 2)^2 + 810  \textbf{The maximum revenue is $810.}

Summary

- **Step 1**
  - Let x = number of price increases/decreases
- **Step 2** (Create two equations)
  - Selling Price = (original price +/- (change in price)) x
  - Number of items sold = #original items +/- (change in number of items)
- **Revenue** R(x) = (Selling price) x (Number of items sold)
- **Step 3** = FOIL the binomials
- **Step 4**: Reorganize into standard form and collect like terms
- **Step 5**: Factor out the coefficient in front of x^2
- **Step 6**: Complete the square (take 1/2 of middle term and square it)
- **Step 7**: Take last term in bracket, multiply by number of front
- **Step 8**: Simplify
- **Step 9**: Go back to Selling price equation and substitute your price increases into the equation if looking for selling price, or read the y value of the vertex for the max/min of the function
Practice Problems

1. Last year, a banquet hall charged $30 per person, and 60 people attended the hockey banquet dinner. This year, the hall’s manager has said that for every 10 extra people that attend the banquet, they will decrease the price by $1.50 per person. What size group would maximize the profit for the hall this year?

2. An electronics store sells an average of 60 entertainment systems per month at an average of $800 more than the cost price. For every $20 increase in the selling price, the store sells one fewer system. What amount over the cost price will maximize profit?
3. A video-game designer has been selling 600 games a week at a selling price of $35 each. He determines that for each dollar the price is lowered, sales will increase by 20 games.
   a) Calculate the selling price that will maximize the income, and calculate the maximum income.
   b) Determine when the income equals zero.