**Maximum/Minimum word problems part 2: Geometry**

**Example 1:**

The owner of a ranch decides to enclose a rectangular region with 140 feet of fencing. To help the fencing cover more land, he plans to use one side of his barn as part of the enclosed region. What is the maximum area the rancher can enclose?

Write an equation to represent the perimeter: \(2w + L = 75\)

Write an equation to represent the area: \(A = w \times L\)

Rearrange the perimeter equation to isolate \(L\)

\[
2w + L = 75
\]

\[
L = 75 - 2w
\]

Then substitute the value of \(L\) into the area equation: \(A = w \times L\)

\[
A = w(75 - 2w)
\]

\[
A = 75w - 2w^2
\]

\[
A = -2w^2 + 75w
\]

\[
A = -2(w^2 - 37.5w + \left(\frac{37.5}{2}\right)^2 - \left(\frac{37.5}{2}\right)^2)
\]

\[
A = -[(w^2 + 37.5w + 351.56) - (351.56)]
\]

\[
A = -(w - 18.75)^2 + 351.56
\]

The largest area is 351.56 m\(^2\)

**Example 2:**

You have a 1200-foot roll of fencing and a large field. You want to make two paddocks by splitting a rectangular enclosure in half. What are the dimensions of the largest such enclosure?

With the labelling I've chosen, the fencing gives me a "perimeter" of \(2L + 3w = 1200\).

Solving for one of the variables, I get:

\[
2L + 3w = 1200
\]

\[
L + 1.5w = 600
\]

\[
L = -1.5w + 600
\]
Then the area is:

\[ A = Lw = (-1.5w + 600)w = -1.5w^2 + 600w \]

To maximize this area, I have to find the vertex. Since all I need are the dimensions (not the area), all I need from the vertex \((h, k)\) is the value of \(h\), since this will give me the maximal width.

**Complete the square**

\[
A = -1.5\left(w^2 - 400w + \left(\frac{400}{2}\right)^2 - \left(\frac{400}{2}\right)^2\right)
\]

\[
A = -1.5\left(w^2 - 400w + 40000\right) - 40000
\]

\[
A = -(w - 200)^2 + 60000
\]

Dimensions are:

\[
w = 200 \text{ m}
\]

\[
L = -1.5w + 600
\]

\[
w = -1.5(200) + 600
\]

\[
L = 300 \text{ m}
\]

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**Summary of steps for Max Area Problems**

- **Step 1**: Create two equations
  - Write an equation for area
  - Write an equation for perimeter
  - Re-write the perimeter equation in terms of \(L\)
- **Step 2**: Substitute the equation of \(L\) into the equation for area
- **Step 3**: Expand the equation
- **Step 4**: Rewrite the equation in standard form
- **Step 5**: Complete the square (take \(\frac{1}{2}\) of middle term and square it)
- **Step 6**: Take last term in bracket, multiply by number of front
- **Step 8**: Simplify
- **Step 9**: The maximum area is taken from the \(k\) value of the vertex. The \(w\) value of the vertex gives the width. The length is determined from the length equation generated in step 1.