Word Problems using the roots of the quadratic equation
(Factored form or Quadratic formula)

Part A: Population Growth Problems

Population growth can be modeled using quadratic functions. In these problems, the output of the function is the number of individuals in the community. The input is the amount of time, in years (usually – bacteria growth is measured in minutes.)

Suppose the population of a town is described by \( P = 14t^2 + 650t + 32000 \), where \( P \) is the population and \( t \) is the time in years, with \( t = 0 \) representing the year 2000.

a. What will the population be in 2035?
Let time at 2035 be \( t = 35 \)

\[
P(35) = 14(35^2) + 650(35) + 32000
\]

\[
P(35) = 71,900
\]

b. When will the population reach 50000?

Solve for \( t \) at \( P(50,000) \)

\[
50,000 = 14t^2 + 650t + 32000
\]

Rearranged

\[
14t^2 + 650t - 18,000 = 0
\]

\[
2(7t^2 + 325t - 9,000) = 0
\]

Too hard to factor, use quadratic formula

\[
a = 7 \quad b = 325 \quad c = -9000
\]

\[
-325 \pm \sqrt{325^2 - 4(7)(-9000)}
\]

\[
2(7)
\]

\[
-325 \pm \sqrt{105625 + 252000}
\]

\[
2(7)
\]

\[
= -325 \pm 590.0
\]

\[
14
\]

\[
t = 18.9 \text{ or } t = \text{negative term (not a solution)}
\]

The Population will be 50000 in 2019
Part B:

Two numbers have a difference of 18. The sum of their squares is 194. What are the numbers?

Let’s define our variables:

\[ x = \text{one of the numbers} \]
\[ y = \text{the other number} \]

\[ x - y = 18 \]
\[ x^2 + y^2 = 194 \]
\[ (18 + y)^2 + y^2 = 194 \]
\[ 324 + 36y + y^2 + y^2 = 194 \]
\[ 2y^2 + 36y + 324 - 194 = 0 \]
\[ 2y^2 + 36y + 130 = 0 \]

Again, we have a substitution situation. Solve the simpler equation for a variable and plug it in to the other equation.

\[ y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ y = \frac{-(36) \pm \sqrt{(36)^2 - 4(2)(130)}}{2(2)} \]
\[ y = \frac{-36 \pm 16}{4} \]
\[ y = -5 \text{ or } -13 \]

\[ x = 18 + (-5) \text{ or } y = 18 + (-13) \]
\[ x = 13 \text{ or } x = 5 \]

So the numbers are \(-5\) and 13 or \(-13\) and 5

Part C: Geometric Word Problems

Remember:

- Draw a picture and Write a “Let” statement
- Write an equation
- Solve the equation (REMEMBER: YOU CAN’T HAVE A NEGATIVE LENGTH)
- Check to see if your solution makes sense
- Re-Read the problem to make sure you answered the question
A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway?

The first thing I need to do is draw a picture.

Since I don’t know how wide the path will be, I’ll label the width as "x".

Let the total width will be $x + 12 + x = 12 + 2x$,

Let the total length will be $x + 16 + x = 16 + 2x$.

Therefore area =

$(12 + 2x)(16 + 2x) = 285$

$192 + 56x + 4x^2 = 285$

Rearrange

$4x^2 + 56x - 93 = 0$

Difficult to factor, so use the quadratic formula

$$x = \frac{-56 \pm \sqrt{(56)^2 - 4(4)(-93)}}{2(4)}$$

$$= \frac{-56 \pm \sqrt{3136 + 1488}}{8}$$

$$= \frac{-56 \pm 68}{8}$$

$$= 15.5, -1.5$$

The width cannot be a negative value, therefore the solution is 1.5

The width of the pathway will be 1.5 meters.